Six Sigma Metrics, Confusion and Resolution

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References

- *Implementing Six Sigma*, Forrest W. Breyfogle III, Wiley, 1999
- *Managing Six Sigma*, Forrest W. Breyfogle III, James M. Cupello, Becki Meadows, Wiley, 2001
- "Bottom-Line Success with Six Sigma," Forrest W. Breyfogle III and Becki Meadows, *Quality Progress,* May 2001

Overview

- The metrics of Six Sigma can be useful or they can be deceiving
  - For example, equations to determine process capability indices are basically very simple; however, these equations are very sensitive to the standard deviation input

- This presentation address the confusion and deception that often accompanies Six Sigma process capability metrics, along with a strategy that can improve the communication of process capability issues.

Usage

- Process capability/performance studies assess a process relative to specification criteria
  - Statisticians often challenge how well commonly used capability indices do this
  - Customers often request these indices when communicating with their suppliers
  - Customers also might set process capability/performance targets and then ask its suppliers for to meet these targets

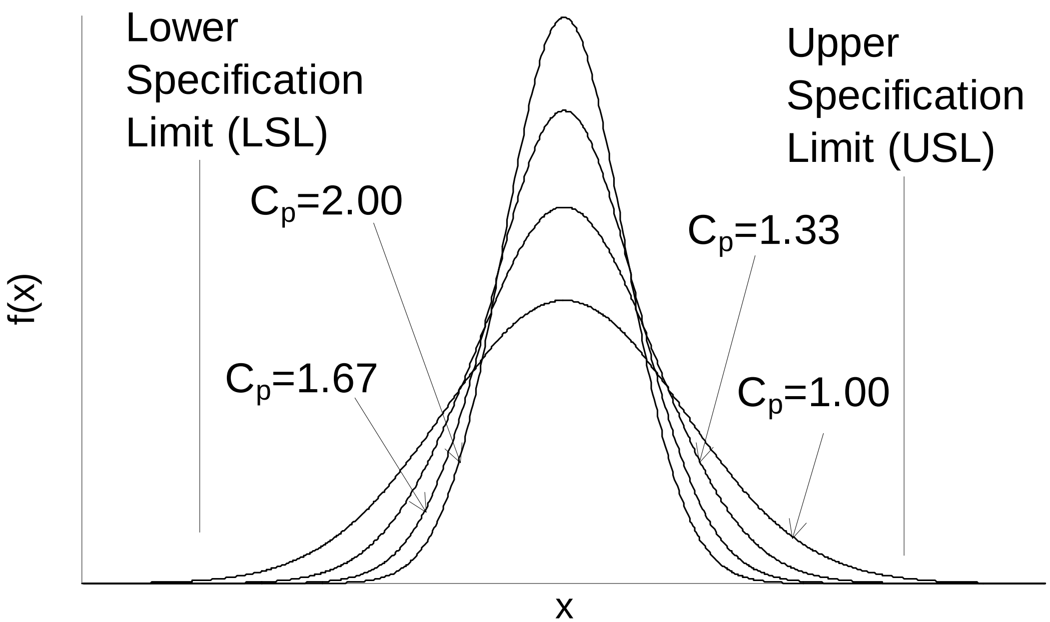
Mathematical Relationships

- Process capability indices C_p and C_{pk} can be expressed as

$$C_p = \frac{\text{USL - LSL}}{6\sigma} \qquad C_{pk} = \min\left[\frac{\text{USL - }\mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right]$$

Mathematical Relationships

- C_p compares the spread of our data to the width of the specification limits



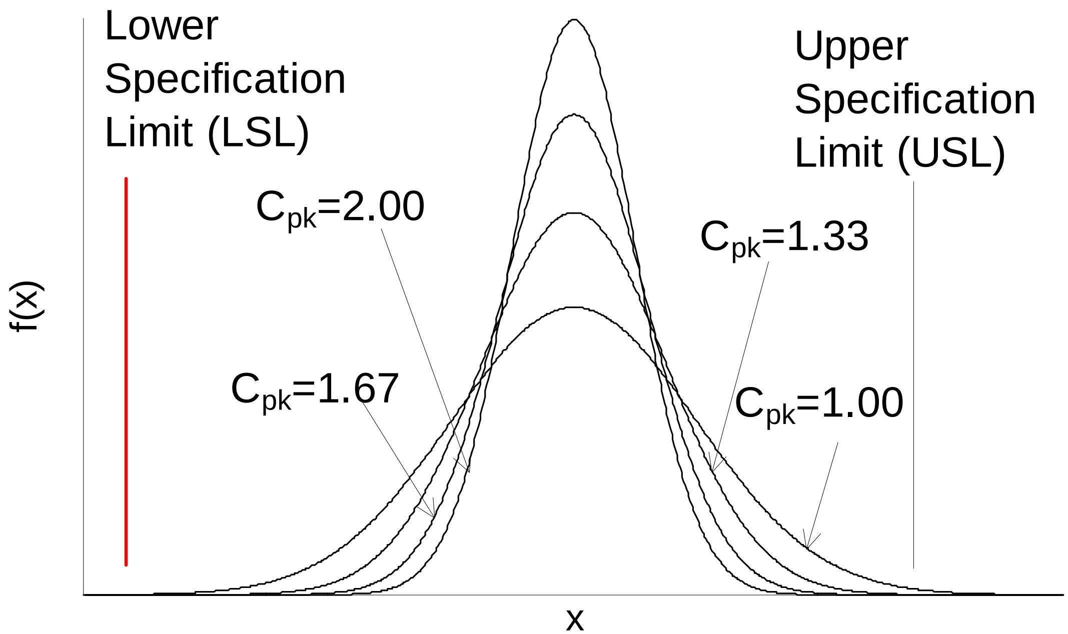
Width of specification limits

$$C_p = \frac{USL - LSL}{6\sigma}$$

Process variability

Mathematical Relationships

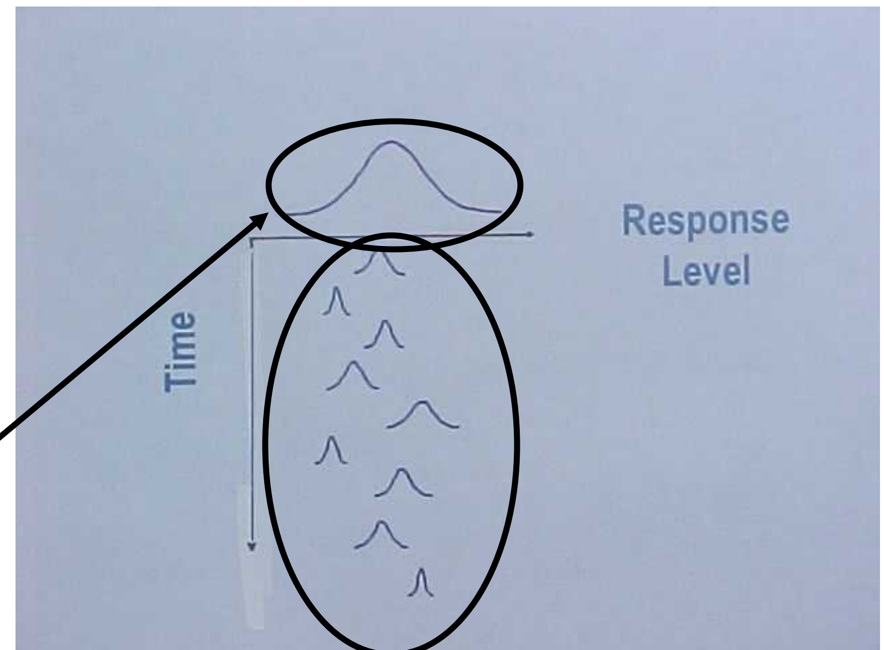
- C_{pk} calculations are made relative to closest specification limit (LSL line shifted left)



$$C_{pk} = \min\left[\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right]$$

Determining Standard Deviation

- Consider how one should determine the standard deviation of a process in order to estimate a process capability index
  - There are differences of opinion on how this should be done
  - The question: Should we focus on
    - overall variability or
    - within subgroup variability



"Average" within standard deviation

Definitions

AIAG Definitions

- Process Capability: The 6σ range of a process' inherent variation, for statistically stable processes only
  - where σ is usually estimated by R-bar/d_2
  - C_p: This is the capability index which is defined as the tolerance width divided by the Process Capability, irrespective of process centering
  - C_{pk}: This is the capability index which accounts for process centering
    - It relates the scaled distance between the process mean and the closest specification limit to half the total process spread

Definitions

AIAG Definitions

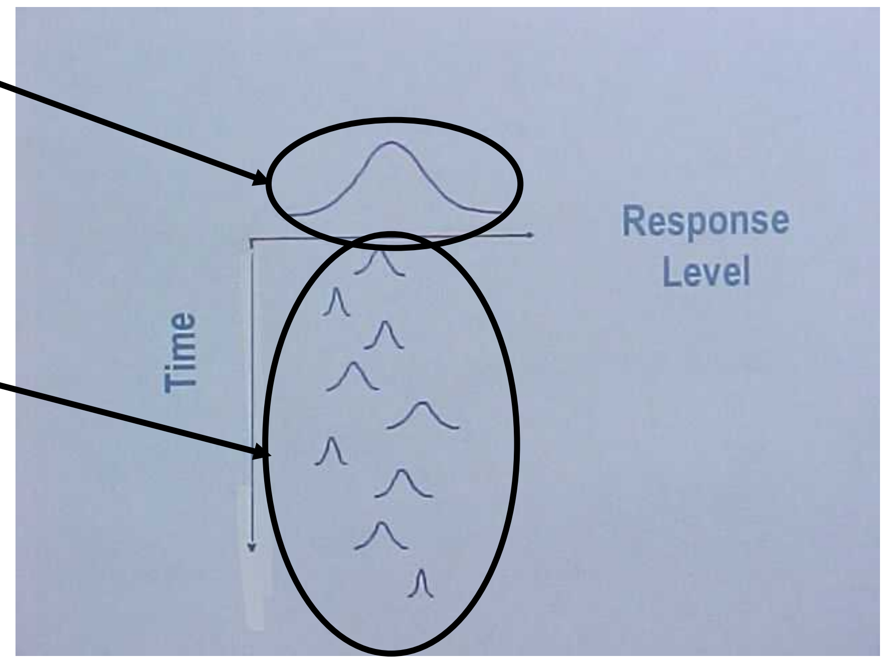
- Process Performance: The 6σ range of a process' total variation,
  - where σ is usually estimated by *s*, the sample standard deviation
  - P_p: This is the performance index which is defined as the tolerance width divided by the process performance, irrespective of process centering
    - Typically, this is expressed as the tolerance width divided by six times the sample standard deviation
    - It should only be used to compare to or with C_p and C_{pk} and to measure and prioritize improvement over time
  - P_{pk}: This is the performance index which accounts for process centering
    - It should only be used to compare to or with C_p and C_{pk} and to measure and prioritize improvement over time

Determining Standard Deviation

- Long-term variability

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}}$$

- Short-term variability
  - if x-bar and R chart: $\hat{\sigma} = \frac{\bar{R}}{d_2}$
  - if XmR chart: $\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{\overline{MR}}{1.128}$



Reported Process Capability Indices

- Not all statistical software programs will give the same answer for process capability calculations
  - Minitab calculations for process capability depend upon the order the data is entered
- The equations described are for normally distributed data
  - Computer programs can often address situations where data are not from a normal distribution; however, this is often not done in practice
- Sample size and confidence intervals are not typically addressed.

Alternative Approach

- Since most people try to visualize the ppm rate impact from any reported process capability index,
  - we suggest reporting only a long-term ppm rate directly using transformations as needed
- We also do not recommend converting ppm rates to Sigma Quality Levels